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Quantum Foam and de Sitter-Like Universe

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We perform a foliation of a four-dimensional Riemannian space-time with respect to a discrete time which is an integer multiple of the Planck time. We find that the quantum fluctuations of the metric have a discrete energy spectrum. The metric field is expanded in stationary eigenstates, and this leads to the description of a de Sitter-like universe. At the Planck scale the model describes a Planckian Euclidean black hole.

1. INTRODUCTION

The concept of a singularity (big bang, black holes, and big crunch) is a classical one, and it is generally assumed that singularities that occur in general relativity can be avoided by a suitable quantum correction to the classical theory.[10,8,5]

There are three kinds of gravitational collapse: black holes, the big crunch, and the "collapse" of space-time at the Planck scale, which is quantum and takes place always and everywhere giving rise to "quantum foam."[21,22,19] Quantum foam is a wormhole-like structure of space that arises at very small scales close to the Planck scale once the huge gravitational quantum fluctuations have destroyed the smooth, simply connected space-time manifold.

Much research work has been done to cure singularities by using quantum effects. One of the most important examples is the Euclidean Schwarzschild black hole,[7,11] where the imaginary time coordinate is identified with a period which is the inverse of the temperature. Another very important example is the Hawking–Hartle quantum cosmology that describes cosmological models with Euclidean metrics: the no-boundary proposal.[9]

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In some current research work[6,1,15,18] the existence of a limiting curvature has been postulated and under this assumption a de Sitter universe, which replaces the singularity, is produced inside a black hole.

In this paper, we consider the third kind of collapse. We make a time slice of 4-dimensional space-time. The initial time and the time step are both equal to the Planck time. The slice n will be at time t_n which is an integer multiple of the Planck time. We look into smaller and smaller regions of space of linear dimension equal to the proper length L_n. The observation requires a large amount of energy: the smaller the region under study, the larger is the energy required to observe it. When we reach the cutoff (the Planck length) we must spend the Planck energy. To "observe" the Planck scale is equivalent to creating a mini black hole with Planck mass.[12] This is just another possible interpretation of the quantum foam.

At the Planck scale, both gravitational and vacuum quantum fluctuations are very strong: in particular, the gravitational quantum fluctuations get their maximal value. Virtual particles and virtual gravitons are created in vacuum, and they can be converted into real particles only if a very large surface gravity is present as in the case of the Hawking radiation.[13]

In this paper, we find that the gravitational quantum fluctuations have a discrete energy spectrum. We then expand the metric field in stationary eigenstates, and this allows us to calculate the expansion factor of an idealized cosmological model based on these assumptions.

The model describes a Euclidean de Sitter-like universe with a positive cosmological constant whose value today is in agreement with inflationary theories. The horizon temperature is the inverse of the period of the gravitational quantum fluctuations.

At the Planck scale the model describes a Planckian Euclidean Schwarzschild black hole.

In Section 2 we consider the Wheeler relation for the quantum fluctuations of the metric in the case of a discrete time slicing of space-time. This leads to the expansion of the metric in stationary eigenstates. In Section 3 we perform a qualitative analysis of the Riemann tensor in terms of the quantum gravitational fluctuations and we find that they have a discrete energy spectrum. In Section 4 we compute the expansion factor for the cosmological model based on the previous results. The model describes a de Sitter-like universe. In Section 5 we consider the static Euclidean form of the metric. We find that there are event horizons at each proper length L_N and that at the Planck scale the de Sitter event horizon coincides with that of a Planckian black hole. In Section 6 we calculate the proper energy E^{Ω} associated to the gradient $\Omega_{N,N+1}$ of the curvature tensor from slice N to slice $N+1$. We find that for N corresponding to the age of the universe, E^{Ω} is the rest mass energy of 10^{80} baryons (the total number of baryons inside the cosmological horizon). Section 7 is devoted to some concluding remarks.

2. STATIONARY EIGENSTATES OF THE METRIC

Let us consider a four-dimensional Riemannian space-time with metric $g_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$). At some point $\bar{x}$ the metric $g_{\mu\nu}(\bar{x})$ has three positive and one negative eigenvalue. Let $g_{\mu\nu}(\bar{x}, t)$ have the generic form

$$g_{\mu\nu}(\bar{x}, t) = \begin{pmatrix} g_{00} = -1 & \underline{0} \\ 0 & g_{ij}(\bar{x}, t) \end{pmatrix}, \qquad i, j = 1, 2, 3 \tag{1}$$

Let us consider a time slicing of space-time. The initial slice is at time $t_0 = t^*$, where t^* is the Planck time:

$$t^* = \left(\frac{\hbar G}{c^5}\right)^{1/2} \cong 5.3 \times 10^{-44} \text{ sec} \tag{2}$$

The time step is $\Delta t = t^*$.

The slice of order n is at time

$$t_n = (n + 1)t^*, \qquad n = 0, 1,2, \ldots \tag{3}$$

To each time t_n corresponds a proper length:

$$L_n = ct_n = (n + 1)L^* \tag{4}$$

where L^* is the Planck length:

$$L^* = ct^* = \left(\frac{\hbar G}{c^3}\right)^{1/2} \cong 1.6 \times 10^{-33} \text{ cm} \tag{5}$$

Let us consider the Wheeler relation[21]

$$\Delta(g_{\mu\nu}) \equiv \frac{\Delta g_{\mu\nu}}{g_{\mu\nu}} \cong \frac{L^*}{L} \tag{6}$$

where $\Delta(g_{\mu\nu})$ is the quantum fluctuation of the metric and L is the linear extension under study. If we take into account the time-slicing (3), Eq. (6) can be written as

$$\Delta(g_{\mu\nu})_n = \frac{L^*}{L_n} = \frac{t^*}{t_n} = \frac{1}{n + 1} \tag{7}$$

For $n = 0$ we recover the well-known result

$$\Delta(g_{\mu\nu})_0 = 1$$

which means that at the Planck scale ($t_0 = t^*$) the quantum fluctuation of the metric takes its maximum value.

Following Wheeler, we take that in quantum geometrodynamics, as well as in electrodynamics, when one examines a region of vacuum of dimension L, the fluctuation energy is of order[21]

$$E \approx \frac{\hbar c}{L} \tag{8}$$

Moreover, in quantum geometrodynamics there is a natural cutoff: the Planck length L^*. In our case, Eq. (8) becomes

$$E_n \cong \frac{\hbar c}{L_n} = \frac{\hbar c}{(n+1)L^*} = \frac{E^*}{n+1} \tag{9}$$

where E^* is the Planck energy:

$$E^* = \left(\frac{\hbar c^5}{G}\right)^{1/2} \cong 1.2 \times 10^{19}\ \text{GeV} \tag{10}$$

Equation (9), as we shall see in more detail in the following, gives the discrete energy spectrum of the gravitational quantum fluctuations.

At the Planck scale ($n = 0$) we recover $E_0 = E^*$.

The proper length L_n in Eq. (9) can be interpreted as the wavelength λ_n associated to the quantum fluctuation Δg_n and Eq. (9) can be written as

$$E_n \cong \frac{\hbar c}{\lambda_n} = \hbar\omega_n = \frac{2\pi\hbar}{P_n} \tag{11}$$

where

$$P_n = 2\pi(n+1)t^* \tag{12}$$

is the period of the gravitational quantum fluctuations.

Let us expand the spatial components of the metric field $g_{ij}(\bar{x}, t)$ in stationary eigenstates $g_{ij_n}(\bar{x}, t)$:

$$g_{ij_n}(\bar{x}, t) = g_{ij_n}(\bar{x})e^{-iE_nt/\hbar}$$

where the eigenvalues E_n are given in Eq. (11).

We get

$$g_{ij}(\bar{x}, t) = \sum_{n=0}^{+\infty} g_{ij_n}(\bar{x})e^{-it/(n+1)t^*} \tag{13}$$

in the infinite sum we do not take into account times preceeding the Planck time.

In Eq. (13) $g_{ij_n}(\bar{x})$ is the 3-geometry of the generic nth hypersurface, t is the continuous time which flows across the slices, and $t_n = (n + 1)t^*$ is the discrete time which labels the slices.

For a fixed $n = N$, the metric field $g_{ij}(\bar{x}, t)$ collapses to the eigenstate N with eigenvalue E_N:

$$g_{ij}(\bar{x}, t) = g_{ij_N}(\bar{x})e^{-iE_N t/\hbar} \tag{14}$$

for $t_N < t < t_{N+1}$, and

$$g_{ij}(\bar{x}, t_N) \equiv g_{ij_N}(\bar{x}) \tag{15}$$

for $t = t_N$.

3. THE DISCRETE ENERGY SPECTRUM

The spatial components of the Riemann tensor at $t = t_N = (N + 1)t^*$ are, by definition, and in dimensional terms only

$$\text{Riem}_N = \frac{g_N^2}{L_N^2}[\alpha(\Delta g_N)^2 + \beta\Delta(\Delta g_N)] \tag{16}$$

where α and β are two real numbers, and $g_N \equiv g_{ijN}(\bar{x})$ stands for any covariant spatial components of the metric tensor at $t = t_N$; that is, g_N is the 3-geometry of the Nth spacelike hypersurface. Here $L_N = (N + 1)L^* \cong g_N^{1/2}\,\Delta x$ is the proper length associated to a general coordinate variation Δx and $\Delta g_N \cong L^*/L_N = 1/(N + 1)$ is the quantum fluctuation of the metric on the slice N.

The terms $(\Delta g_N)^2$ and $\Delta(\Delta g_N)$ in Eq. (16) stem respectively from the terms $\Gamma\Gamma$–$\Gamma\Gamma$ and $\partial\Gamma$–$\partial\Gamma$ in the Riemann tensor.

We obtain

$$(\Delta g_N)^2 \approx \Delta(\Delta g_N) \approx \frac{1}{(N + 1)^2} \tag{17}$$

Then Riem_N in Eq. (16) takes the form

$$\text{Riem}_N \cong \gamma\frac{g_N^2}{(N + 1)^4 L^{*2}} \tag{18}$$

where γ is a constant factor.

The Ricci tensor is defined as

$$\text{Ricci}_N \cong g_N^{-1}\,\text{Riem}_N$$

where g_N^{-1} stands for any contravariant spatial component of the metric tensor at $t = t_N$.

The curvature scalar is defined as

$$R_N \cong g_N^{-2}\ \mathrm{Riem}_N$$

Hence the Einstein tensor

$$G_N = \mathrm{Ricci}_N - \frac{1}{2} R_N G_N$$

has the structure

$$G_N \cong g_N^{-1}\ \mathrm{Riem}_N \cong \gamma \frac{g_N}{(N+1)^4 L^{*2}} \tag{19}$$

The Einstein field equations are

$$G_N - \Lambda_N g_N = \frac{8\pi G}{c^4} T_N \tag{20}$$

where Λ_N is a positive cosmological constant and T_N stands for any spatial components of the stress energy-momentum tensor at $t = t_N$.

Let us integrate the field equations over a 3-dimensional spacelike hypersurface Σ with unit normal $n \cong g_N^{-1/2}$:

$$\int_\Sigma G_N(n, n)\, d\Sigma = \int_\Sigma (\Lambda_N g_N)(n, n)\, d\Sigma + \frac{8\pi G}{c^4} \int_\Sigma T_N(n, n)\, d\Sigma$$

where $d\Sigma$ is the proper volume element of Σ. We get

$$[g_N^{-1} G_N] L_N^3 = [\Lambda_n] L_N^3 + \frac{8\pi G}{c^4} E_N$$

where E_N is the proper energy within a subset of Σ with proper volume L_N^3, and the bracketed quantities are averages over the proper volume of integration. Then, we have

$$\gamma \frac{L^*}{N+1} = \Lambda_N L_N^3 + \frac{8\pi G}{c^4} E_N = \frac{8\pi G}{c^4} [E_N^{\mathrm{VAC}} + E_N] \tag{21}$$

where

$$E^{\mathrm{VAC}} = \int_\Sigma T_N^{\mathrm{VAC}}(n, n)\, d\Sigma = \frac{c^4}{8\pi G} \Lambda_N L_N^3$$

For $\Lambda_N = 0$, we get

$$E_N = \gamma \frac{c^4}{8\pi G} \frac{L^*}{N+1} = \gamma \frac{c^5 t^*}{8\pi G} \frac{1}{N+1} = \frac{\gamma}{8\pi} \frac{E^*}{N+1} \tag{22}$$

where E^* is the Planck energy in Eq. (10) and the time–energy uncertainty principle has been used with $\Delta t = t^*$.

The proper energy E_N in Eq. (22) is the quantized gravitational energy carried by the quantum fluctuation of the metric on the Nth slice.

From the interpretation of the Landau–Liftshitz pseudotensor $\hat{t}_N$, it follows that the nonlocal gravitational energy resides in the nonlinear terms of the curvature tensor. By definition we have

$$\left|\det g_N\right| g_N^{-2}\, \hat{t}_N \cong \left(\frac{\Delta g_N}{\Delta x}\right)^2 \tag{23}$$

where $\left|\det g_N\right| \cong g_N^4$. Then we have

$$\hat{t}_N \cong \frac{1}{g_N^2} \left(\frac{\Delta g_N}{\Delta x}\right)^2 \tag{24}$$

By integration, we get

$$\int_\Sigma \hat{t}_N(n, n)\, d\Sigma = [g^{-1} \hat{t}_N] \cong \frac{8\pi G}{c^4} \frac{E_N^{\mathrm{GRAV}}}{L_N^3} \tag{25}$$

From Eqs. (24) and (25) it follows that

$$\left[\frac{1}{g_N^3} \left(\frac{\Delta g_N}{\Delta x}\right)^2\right] \cong \frac{8\pi G}{c^4} \frac{E_N^{\mathrm{GRAV}}}{L_N^3}$$

where $1/(\Delta x)^2 = g_N / L_N^2$. We obtain from Eq. (25)

$$\left(\frac{\Delta g_N}{g_N}\right)^2 \cong \frac{8\pi G}{c^4} \frac{E_N^{\mathrm{GRAV}}}{(N+1) L^*}$$

where we recall that $(\Delta g_N / g_N)^2 = 1/(N+1)^2$.

Finally we have

$$E_N^{\mathrm{GRAV}} \cong \frac{E^*}{N+1} \tag{26}$$

For $\Lambda_N \neq 0$, the proper energy is shared between vacuum and gravitational quantum fluctuations:

$$E_N^{\mathrm{VAC}} + E_N^{\mathrm{GRAV}} = \gamma \frac{c^4}{8\pi G} \frac{L^*}{N+1} \tag{27}$$

4. THE COSMOLOGICAL MODEL

Our purpose is to build up an isotropic and homogeneous cosmological model based on the previous assumptions. We shall use the Robertson–Walker metric. For simplicity we consider the flat case ($K = 0$):

$$ds^2 = -dt^2 + R^2(t)[dr^2 + r^2\, d\Omega^2] \tag{28}$$

where $d\Omega^2 = d\vartheta^2 + \sin^2\vartheta\, d\varphi^2$ is the surface element of the unit 2-sphere and $R(t)$ is the expansion factor.

Our aim is to compute $R(t)$. The first step is to factorize the initial 3-geometry $g_{ij0}\,(\bar{x})$ in Eq. (13). At cosmological scales (very large N) higher order fluctuations of the metric can be discarded and we get

$$g_N \approx (N+1)g_0 \tag{29}$$

where g_0 is the 3-geometry of the initial slice at $t = t_0 \equiv t^*$. By the use of Eq. (29), Eq. (13) becomes

$$g_{ij}(\bar{x}, t) = g_{ij0}(\bar{x}) \sum_{n=0}^{N} (n+1)\, e^{-it/(n+1)t^*} \tag{30}$$

Let us consider the line element

$$d\sigma^2 = g_{ij}(\bar{x}, t)\, dx^i\, dx^j$$

and let us assume that we know the initial 3-geometry at the initial time $t = t_0$,

$$\gamma_{ij}(\bar{x}) \equiv g_{ij}(\bar{x}, t_0)$$

Two adjacent world lines of the cosmological fluid with coordinates respectively $\{x^i\}$ and $\{x^i + \Delta x^i\}$ at time t_0 are separated by the proper distance

$$\Delta\sigma(t_0) = (\gamma_{ij}\, \Delta x^i\, \Delta x^j)^{1/2}$$

At some later time t, the two word lines will be separated by some other proper distance $\Delta\sigma(t)$. The spatial constant ratio $\Delta\sigma(t)/\Delta\sigma(t_0)$ defines the expansion factor $R(t)$.

In our case

$$\gamma_{ij}(\bar{x}) \equiv g_{ij0}(\bar{x}), \qquad t_0 = t^*$$

and $g_{ij}(\bar{x}, t)$ is given by Eq. (30). Then we have

$$\Delta\sigma(t^*) = (g_{ij0}(\bar{x})\, dx^i\, dx^j)^{1/2} \tag{31}$$

$$\Delta\sigma(t) = \left(g_{ij0}(\bar{x}) \sum_{n=0}^{N} (n + 1)e^{-it/(n+1)t^*}\, dx^i dx^j \right)^{1/2} \tag{32}$$

and the expansion factor takes the form

$$R(\tau) = \frac{\Delta\sigma(t)}{\Delta\sigma(t^*)} = \sum_{n=0}^{N} (n + 1)^{1/2}\, e^{\tau/2(n+1)t^*} \tag{33}$$

where $\tau = -it$.

The time derivative of the expansion factor is

$$\dot{R}(t) = \frac{1}{2t^*} \sum_{n=0}^{N} e^{\tau/2(n+1)t^*} \tag{34}$$

Numerical computations give the expression of the Hubble constant for large N (and $0 < \tau < 1$)

$$H_N = \frac{\dot{R}(\tau)}{R(\tau)} = \frac{3}{2(N + 1)t^*} \tag{35}$$

The cosmological time is defined as

$$H_N^{-1} = \frac{2}{3}(N + 1)t^* \tag{36}$$

By the use of the expression of $R(t)$ in Eq. (33), the Robertson–Walker metric in Eq. (31) becomes

$$ds^2 = -dt^2 + \sum_{n=0}^{N}(n + 1)e^{-it/(n+1)t^*}\, [dr^2 + r^2\, d\Omega^2] \tag{37}$$

The field equations are those of a de Sitter-like model:

$$3H_N^2 = \Lambda_N c^2 \tag{38}$$

By the use of Eq. (35), we get from Eq. (38)

$$\Lambda_N = \frac{27}{4} \frac{1}{(N + 1)L^{*2}} \qquad \text{(for large } N\text{)} \tag{39}$$

The vacuum energy density is then

$$\rho_N^{\mathrm{VAC}} = \frac{c^2\Lambda_N}{8\pi G} = \frac{27c^2}{32\pi G} \frac{1}{(N + 1)^2 L^{*2}} \tag{40}$$

The cosmological time today is $H^{-1} \cong 5 \times 10^{17}$ sec, which corresponds to the value $N \cong 7 \times 10^{60}$ in Eq. (36).

From Eq. (39) it follows that the value of the cosmological constant today is

$$\Lambda_{\mathrm{NOW}} \cong 5 \times 10^{-56}\ \mathrm{cm}^{-2} \tag{41}$$

which is in agreement with inflationary theories.

At the Planck scale ($n = 0$), one verifies that

$$H_0 = \frac{1}{2t^*} \tag{42}$$

and

$$\Lambda_0 \approx R^* \tag{43}$$

where $R^* = 1/L^{*2}$ is the radius of Planckian curvature.

5. THE PLANCKIAN BLACK HOLE PARADIGM

Let us consider the static form (with Killing vector $\zeta = \partial t$) of the de Sitter metric:

$$ds^2 = -\left(1 - \frac{H^2}{c^2} r^2\right) dt^2 + \left(1 - \frac{H^2}{c^2} r^2\right)^{-1} dr^2 + r^2\, d\Omega^2 \tag{44}$$

which is rather like the Schwarzschild solution:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2 \tag{45}$$

In Eq. (44) there is an apparent singularity at $r = c/H$ that can be removed (as in the Schwarzschild case) by a coordinate transformation giving rise to an event horizon.

In our model, there are event horizons at

$$r_N = \frac{3}{2} \frac{c}{H_N} = L_N = (N + 1)t^* \tag{46}$$

where H_N is given by Eq. (35) for large N and by Eq. (42) for $n = 0$.

However, while a black hole has an absolute horizon, the de Sitter horizon is an observer-dependent horizon. This means that an observer on the Nth hypersurface at $t = t_N$ will receive signals from all the other hypersurfaces at $t = t_n$ with $n < N$, but not from those with $n > N$. Only at the Planck scale ($n = 0$) does the de Sitter horizon

$$r_0 = \frac{c}{H_0} = 2L^* \tag{47}$$

become an absolute horizon, as it coincides with the Schwarzschild radius of a Planckian black hole:

$$r^* = \frac{2M^*G}{c^2} = 2L^* \tag{48}$$

where M^* is the Planck mass:

$$M^* = \left(\frac{\hbar c}{G}\right)^{1/2} \cong 2.17 \times 10^{-5}\ \text{g} \tag{49}$$

If in the static form of the de Sitter metric (44) we put $\tau = -it$, we get a Euclidean metric[11]

$$ds^2 = d\tau^2 + \frac{1}{H^2}\cos H\,\tau(dr^2 + \sin^2 r\, d\Omega^2)$$

In our model, there is an apparent singularity at $H_N = 0$ (for $N \to +\infty$) that can be removed as in the most general case discussed by Hawking.

The imaginary time is identified with the period

$$P_N = \frac{3}{2}\frac{2\pi}{H_N} = 2\pi(N+1)t^* \tag{50}$$

Let us recall that the period P_N in Eqs. (12) and (50) is the period of the gravitational quantum fluctuations.

The temperature is

$$T_N = \frac{1}{P_N} = \frac{1}{2\pi(N+1)t^*} \tag{51}$$

the area of the event horizon is

$$A_N = \frac{9}{4}\frac{4\pi c^2}{H_N^2} = 4\pi L_N^2 \tag{52}$$

and the entropy is

$$\sigma_N = \frac{9}{4}\frac{\pi c^2}{H_N} = \pi L_N^2 = \frac{1}{4} A_N \tag{53}$$

At the Planck scale ($n = 0$) we have

$$P_0 = 2\pi t^* \tag{54}$$

$$T_0 = \frac{1}{P_0} = \frac{1}{2\pi t^*} \tag{55}$$

$$A_0 = 4\pi L^{*2} \tag{56}$$

$$\sigma_0 = \pi L^{*2} = \frac{1}{4} A_0 \tag{57}$$

At the Planck scale, the expansion factor in Eq. (33) is the generator of a $U(1)$ group:

$$R(t) = e^{-it/2t^*} \tag{58}$$

This makes space-time at the Planck scale a multiply connected manifold.

The space-time topology at the Planck scale becomes the topology of a Euclidean Schwarzschild black hole:

$$R_{\mathrm{SPACE}} \times S^1_{\mathrm{TIME}} \times S^2 \tag{59}$$

This interpretation of the quantum foam does not disagree with that of Wheeler; in fact, under a dual transformation

$$U(1)_{\mathrm{TIME}} \Leftrightarrow U(1)_{\mathrm{SPACE}} \tag{60}$$

The space-time topology in Eq. (59) becomes that of a wormhole with one handle[2]

$$R_{\mathrm{TIME}} \times S^1_{\mathrm{SPACE}} \times S^2 \tag{61}$$

We conclude by saying that at the Planck scale our cosmological model behaves as a Planckian black hole. So at that scale the Hawking effect[13] may occur because of the high temperature.

6. THE BARYON NUMBER

The gradient of the gravitational quantum fluctuations from slice N to slice $N + 1$ is

$$\omega_{N,N+1} = \Delta g_N - \Delta g_{N+1} = \frac{1}{(N+1)(N+2)} \tag{62}$$

and the gradient of the curvature tensor from slice N to slice $N + 1$ is defined as

$$\Omega_{N,N+1} = \frac{g_N g_{N+1}}{L_N L_{N+1}} \omega_{N,N+1} = \frac{g_0^2}{L^{*2}} \frac{1}{(N+1)(N+2)} \tag{63}$$

We recall that

$$g_N = (N+1) g_0, \qquad L_N = (N+1) L^* \tag{64}$$

where L^* is the Planck length and g_0 is the three-geometry on the initial slice

$N = 0$. The $\Omega_{N,N+1}$ in Eq. (63) is responsible for tidal effects which occur in the change of the radius $L_N \to L_{N+1}$ of the event horizon of the de Sitter universe. At very small scales, the tidal effects are very large. For $N = 0$, where the de Sitter event horizon coincides with the Planckian black hole horizon, the Hawking radiation takes place.

A stress energy-momentum tensor $T_N^{(\Omega)}$ can be associated to $\overline{\Omega}_{N,N+1}$ where $\overline{\Omega}_{N,N+1} \equiv g_0^{-1}\,\Omega_{N,N+1}$, by the field equations:

$$\overline{\Omega}_{N,N+1} = \frac{8\pi G}{c^4}\,T_N^{(\Omega)} \tag{65}$$

Let us integrate Eq. (65) over a 3-dimensional hypersurface Σ with unit normal $n \cong g_0^{-1/2}$:

$$\int_\Sigma \overline{\Omega}_{N,N+1}\,(n,n)\,d\Sigma = \frac{8\pi G}{c^4}\int_\Sigma T_N^{(\Omega)}\,(n,n)\,d\Sigma \tag{66}$$

where $d\Sigma$ is the proper volume element of Σ. We get

$$[g_0^{-1}\,\overline{\Omega}_{N,N+1}]\,L_N^3 = \frac{8\pi G}{c^4}\,E_N^{(\Omega)} \tag{67}$$

where $E_N^{(\Omega)}$ is the proper energy within a subset of Σ with proper volume L_N^3 and the bracketed quantity is an average over the proper volume of integration.

Then, we have

$$E_N^{(\Omega)} = \frac{c^4}{8\pi G}\,\frac{(N+1)^2}{N+2}\,L^* = \frac{1}{8\pi}\,\frac{(N+1)^2}{N+2}\,E^* \tag{68}$$

where E^* is the Planck energy.

For $N \cong 7 \times 10^{60}$ corresponding to the cosmological time $H_{\mathrm{NOW}}^{-1} = 5 \times 10^{17}$ sec, we obtain

$$E_{\mathrm{NOW}}^{(\Omega)} \approx 10^{80} m_p c^2 \tag{69}$$

where $m_p \cong 1.6 \times 10^{-24}$ g is the proton mass and the number $10^{80} \approx N_E$ is the total number of baryons inside the visible universe. The number $N_E = 4/3\,\pi(H_{\mathrm{NOW}}^{-1}c)^3\rho_{\mathrm{NOW}}/m_p \approx 10^{79}$ (where H_{NOW}^{-1} is the age of the universe, ρ_{NOW} is the present density of the universe, m_p is the proton mass, and c is the velocity of light) was evaluated by Eddington[4] and is often termed the "Eddington number."

Then we can interpret the proper energy in Eq. (69) as the rest mass energy of the total number of baryons inside the cosmological horizon.

7. CONCLUSION

In this cosmological model, we have considered the Planck time as the quantum of time in performing the time slicing. This has been done with the aim of describing the behavior of the quantum fluctuations of the metric from the Planck scale to the Hubble radius scale. Then the model describes a Planckian Euclidean black hole which suddenly turns into a Euclidean de Sitter-like universe. We interpret this quick phase transition as due to the fact that the Planckian black hole emits Hawking radiation very rapidly. In fact, Hawking radiation rate is related to the gradient of the curvature tensor $\Omega_{N,N+1}$ and it decreases down quite rapidly, as $\exp(-N^2)$. The sudden evaporation makes the black hole cool and expand (the temperature falls as $1/N$ and the area increases as N^2).

At this point one could wonder whether the model describes a Planckian black hole that expands to a de Sitter universe, or, conversely, a de Sitter universe that implodes to a Planckian black hole. The answer is given by the arrow of time, defined by the increase of entropy. In Section 5 we found that entropy increases as N^2, thus the model should describe an expanding Planckian black hole.

Penrose's Weyl tensor hypothesis (WTH)[16] is a speculation that the Weyl tensor may be related to a sort of gravitational entropy which is zero for an initial singularity and goes to infinity for a final singularity. This would account for the time asymmetry of the universe.[17] In our model, we interpret $[(\Delta g)_N]^{-2} \approx (N+1)^2$ as the gravitational entropy of the Nth hypersurface.

Apparently, an inconsistency with the WTH arises in this model. In fact, it would seem unlikely that a final singularity with infinite Weyl curvature (in our case the Planckian black hole) could also be an initial singularity with zero Weyl curvature (in our case the starting point of the de Sitter universe) unless strong cosmic censorship fails, at least at the Planck scale. The fact that quantum effects could violate the cosmic censorship hypothesis has also been discussed by Hawking.[14]

We recall that we are dealing with a Planckian black hole and a de Sitter universe that are both Euclidean, so that they are supposed to be singularity-free. Nevertheless, we guess that some relation must exist between the failure of the strong cosmic censorship at the Planck scale and the $U(1)$ dual symmetry described in Section 5.

Moreover, for an initial singularity which has zero Weyl curvature, all the curvature must reside in the Ricci tensor, so that matter must be present from the beginning and "creation *ex nihilo*"[20] should be excluded.

In our case, however, the model is that of a Euclidean de Sitter-like universe (with no "starting point") which is empty although the Ricci tensor is not identically zero. In fact, in this model, the Ricci tensor is the "effective energy-momentum tensor" of the geometry quantum fluctuations.

To get a more realistic model with matter, one should take into account the gradient of the curvature tensor $\Omega_{N,N+1}$, which can be interpreted as a modification term of Einstein's field equations at the Planck scale due to the huge vacuum and geometry quantum fluctuations. The proper energy E_N^{Ω} that is associated to $\Omega_{N,N+1}$ increases with N. The Hawking radiation that reaches an observer on the Nth hypersurface is then interpreted as consisting of the total number of particles inside the horizon at the cosmological time H_N^{-1}, with rest mass energy E_N^{Ω}. (Perhaps we should recall that, as the de Sitter horizon is observer-dependent, the Nth hypersurface will receive signals from all the other n-hypersurfaces, with $n < N$.)

The present value of E_N^{Ω} (for $N \approx 10^{60}$) is the rest mass energy of 10^{80} baryons, in agreement with Eddington's arguments. This result seems to be related to "the large number hypothesis" (LNH) that originally goes back to Eddington and is concerned with the fact that there is a numerical coincidence between various basic quantities in cosmology, including the baryon number. The LNH was developed and made formal by Dirac, (3) who further required that Newton's gravitational constant G varies with time. Of course, as is well known, this theory failed against experiment. What remains at present of the LNH is the anthropic principle approach, which is primarily philosophical. We hope that our result could be a hint at a modern mathematical interpretation of the LNH. In fact, Eddington originally conceived the LNH in the perspective that those numerical coincidences could be explained by theoretical arguments.

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